Reduced Row Echelon Form

> *Definition:* A matrix is said to be in reduced row echelon form if
>
> 1. It is in *row echelon form.*
> 2. The first nonzero entry in each row is 1.
> 3. The first nonzero entry in each row is the only nonzero entry in its column.

> *Theorem 2:* Every matrix is row equivalent to a *unique* matrix in reduced row echelon form (RREF).

Example 5: Write the given augmented matrix in reduced row echelon form.

$$\left[\begin{array}{ccccc|c} 1 & -2 & 0 & 2 & -3 & 1 \\ 0 & 0 & 0 & 2 & -4 & -2 \\ -2 & 4 & -1 & -2 & 2 & -4 \end{array}\right] \tag{1}$$

Part 1

$$\left[\begin{array}{ccccc|c} 1 & -2 & 0 & 2 & -3 & 1 \\ 0 & 0 & 0 & 2 & -4 & -2 \\ -2 & 4 & -1 & -2 & 2 & -4 \end{array}\right] \quad R_3 := R_3 + 2R_1 \quad \left[\begin{array}{ccccc|c} 1 & -2 & 0 & 2 & -3 & 1 \\ 0 & 0 & 0 & 2 & -4 & -2 \\ 0 & 0 & -1 & 2 & -4 & -2 \end{array}\right] \quad R_3 \leftrightarrow R_2$$

Part 2

$$\sim \left[\begin{array}{ccccc|c} 1 & -2 & 0 & 2 & -3 & 1 \\ 0 & 0 & -1 & 2 & -4 & -2 \\ 0 & 0 & 0 & 2 & -4 & -2 \end{array}\right] \quad \begin{array}{l} R_3 := \frac{1}{2} R_3 \\ R_2 := -R_2 \end{array} \quad \left[\begin{array}{ccccc|c} 1 & -2 & 0 & 2 & -3 & 1 \\ 0 & 0 & 1 & -2 & 4 & 2 \\ 0 & 0 & 0 & 1 & -2 & -1 \end{array}\right]$$

Part 3

$$R_1 := R_1 - 2R_3$$
$$R_2 := R_2 + 2R_3$$

$$\boxed{\sim \left[\begin{array}{ccccc|c} 1 & -2 & 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & -1 \end{array}\right]}$$